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A NOTE ON FINITE DIFFERENCE ESTIMATION OF ACOUSTIC PARTICLE VELOCITY

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The influence of instrumentation amplitude and phase mismatch on finite difference measurement of mean square particle velocity is examined. It is shown theoretically and demonstrated experimentally that the resulting error depends on two properties of the sound field, the ratio of the active intensity to the mean square particle velocity and the ratio of the reactive intensity to the mean square particle velocity. Very large errors can occur near particle velocity nodes in interference fields even with well-matched state-of-the-art sound intensity measurement systems. However, outside of such regions, moderate phase mismatch has very little influence on the performance of the measurement system. Amplitude mismatch is somewhat more problematic.

1. INTRODUCTION

Pressure sensing condenser microphones are more stable and reliable than any other acoustic transducers, but it is sometimes useful to measure the particle velocity. The idea of measuring the total energy density in a sound field rather than just the potential energy density (or the squared sound pressure) goes back to the early 1930s [1]; this involves determining the sound pressure and three perpendicular components of the particle velocity. One advantage, demonstrated experimentally in reference [1] and analyzed theoretically in reference [2], is that the total energy density tends to vary less with the position in a reverberant enclosure than the potential energy density. Possible applications of energy density sensors include active noise control systems, since minimizing the total energy density at a point in an enclosure is a more efficient control strategy than minimizing the sound pressure [3]. Because of the directional information, measurements of the particle velocity have also found applications in room acoustics [4], as already suggested in reference [1]. Yet another application is in analyzing large, complicated sources of noise; a small omnidirectional source with a strength that is measured on-line would be very useful in reciprocity experiments for characterizing machinery noise sources [5, 6]. However, it is more difficult to measure the particle velocity than the sound pressure. The particle velocity is usually estimated using a finite difference arrangement of pressure microphones.

Finite difference estimation involves approximating gradients with differential quotients. For example, the "two-microphone" method of measuring sound intensity is based on approximating the gradient of the sound pressure in the axial direction of the measurement probe, $\partial p/\partial r$, with the ratio $\Delta p/\Delta r$, where Δp is the difference between the two microphone signals and Δr is the microphone separation distance. It is well known that sound-intensity estimates based on this measurement principle are subject to errors from many sources [7].

Estimation of particle velocity would seem to be more difficult, since not only phase but also amplitude mismatch is a problem. The most important errors and limitations in finite difference estimation of particle velocity would seem to be caused by (i) the finite difference approximation itself, (ii) instrumentation phase mismatch, (iii) amplitude (sensitivity) mismatch, (iv) the finite vent sensitivity of the microphones, (v) electrical noise, and (vi) interference of the microphones on the sound field.

Most of these problems have been dealt with in the literature. The finite difference error has been analyzed thoroughly in references [7, 8]. The influence of the microphone pressure equalization vents, which is significant in sound-intensity measurement with inexpensive electret microphones, has, perhaps surprisingly, been shown to be negligible in measurement of the particle velocity [9]. As to the influence of electrical noise, it is a simple matter to show that the signal-to-noise ratio compared with single-microphone measurement of the sound pressure is reduced by $10\log(2/(k\Delta r)^2)$ dB, where k is the wavenumber (see Appendix A). Interference/diffraction effects obviously depend on the geometry of the sensor; such effects have been examined in reference [10] for the special case of a spherical sensor.

This note concentrates on the influence of phase and amplitude mismatch. Several recent papers have examined these sources of error in finite difference measurements of the total energy density of sound fields [8, 10, 11]. However, the resulting expressions are quite complicated even for simple one-dimensional sound fields [8], making it somewhat difficult to draw general conclusions. The purpose of this paper is to supplement the error studies presented in references [8, 10, 11] with some simple considerations making use of findings from the literature on sound intensity.

2. OUTLINE OF THEORY

The sound pressure in a harmonic sound field can be written in the form

$$p = |p|e^{j\varphi}. \tag{1}$$

It now follows from Euler's equation of motion (with the $e^{j\omega t}$ convention) that the particle velocity can be written as

$$j\omega\rho\mathbf{u} = -\nabla p = -\nabla(|p|e^{j\varphi}) = -j|p|e^{j\varphi}\nabla\varphi - e^{j\varphi}\nabla|p|, \tag{2}$$

where ρ is the density of the medium. Therefore, the squared amplitude of the particle velocity at a given point in a sound field can be expressed in terms of the gradient of the phase of the sound pressure and the gradient of the amplitude of the sound pressure as follows:

$$|\mathbf{u}|^2 = \frac{|p|^2(\nabla\varphi)^2 + (\nabla|p|)^2}{(\rho\omega)^2}. \tag{3}$$

Since the active sound intensity is proportional to the gradient of the phase of the pressure [12, 13],

$$\mathbf{I} = \tfrac{1}{2}\mathbf{Re}\{p\mathbf{u}^*\} = -\frac{|p|^2}{2\omega\rho}\nabla\varphi, \tag{4}$$

and the reactive intensity is proportional to the gradient of the amplitude of the pressure [13, 14],

$$\mathbf{J} = \tfrac{1}{2}\,\mathrm{Im}\{p\mathbf{u}^*\} = -\frac{|p|\nabla|p|}{2\omega\rho} = -\frac{\nabla|p|^2}{4\omega\rho}, \tag{5}$$

the squared amplitude of the particle velocity can also be written in the form

$$|\mathbf{u}|^2 = 4\,\frac{|\mathbf{I}|^2 + |\mathbf{J}|^2}{|p|^2}, \tag{6}$$

in agreement with the fact that

$$|p|^2|\mathbf{u}|^2 = 4(|\mathbf{I}|^2 + |\mathbf{J}|^2) \tag{7}$$

in a pure tone sound field [15].

In the following, the influence of phase and amplitude mismatch on measurement of the particle velocity is analyzed, use being made of equation (6) and results from the literature on measurement of sound intensity. For simplicity, the analysis is limited to particle velocity components in one direction only.

2.1. ERRORS DUE TO PHASE AND AMPLITUDE MISMATCH

It can be deduced from equation (4) that finite difference estimates of the active sound intensity are seriously affected by phase mismatch between the two measurement channels, whereas amplitude mismatch has very little influence. It is a common practice to express the error due to phase mismatch in terms of the residual intensity I_o as follows,

$$\hat{I}_r = I_r + I_o(\overline{p^2}/\overline{p_o^2}), \tag{8}$$

where ^ indicates a biased estimate of the sound-intensity component I_r, and I_o is the "false" active intensity indicated by the measurement system because of a phase error φ_e when the two microphones of the measurement system are exposed to the same sound pressure, p_o, in a small coupler [16, 17],

$$I_o = -\frac{\overline{p_o^2}}{\omega\rho}\frac{\varphi_e}{\Delta r} = -\frac{\overline{p_o^2}}{\rho c}\frac{\varphi_e}{k\Delta r}. \tag{9}$$

Evidently, this quantity should be as small as possible.

Finite difference estimates of the reactive sound intensity are seriously affected by amplitude mismatch, but not at all by phase mismatch, as can be seen from equation (5). By analogy with equation (8), one may express the biased estimate of the reactive intensity in terms of the residual reactive intensity J_o as follows:

$$\hat{J}_r = J_r + J_o(\overline{p^2}/\overline{p_o^2}), \tag{10}$$

where J_o is the "false" reactive intensity indicated by the measurement system because of amplitude mismatch when the two microphones are exposed to the same sound pressure, p_o, in a small coupler [18],

$$J_o = -\frac{\overline{p_o^2}}{2\omega\rho}\frac{\delta_e}{\Delta r} = -\frac{\overline{p_o^2}}{2\rho c}\frac{\delta_e}{k\Delta r}. \tag{11}$$

The quantity δ_e is the fractional amplitude mismatch, that is, the ratio of the sensitivities of the two microphones is $(1 + \delta_e)$, corresponding to a sensitivity error of 20log $(1 + \delta_e) \simeq 8{\cdot}6\delta_e$ dB.

Equations (8) and (10) demonstrate that the error in I_r and J_r depends on the local ratio of the mean square pressure to I_r and J_r respectively. Combining with equation (6) gives the following expression for the biased estimate of the mean square particle velocity:

$$\widehat{\overline{u_r^2}} = \frac{(\hat{I}_r^2 + \hat{J}_r^2)}{\overline{p^2}}$$

$$= \overline{u_r^2}\left(1 + \frac{I_o}{\overline{p_o^2}/\rho c}\frac{2I_r}{\rho c\overline{u_r^2}} + \frac{J_o}{\overline{p_o^2}/\rho c}\frac{2J_r}{\rho c\overline{u_r^2}} + \left(\left(\frac{I_o}{\overline{p_o^2}/\rho c}\right)^2 + \left(\frac{J_o}{\overline{p_o^2}/\rho c}\right)^2\right)\frac{\overline{p^2}}{(\rho c)^2\overline{u_r^2}}\right). \qquad (12)$$

Increasing the microphone separation distance reduces I_o and J_o and thus the error due to phase and amplitude mismatch, at the expense of increasing the finite difference error and thus reducing the upper frequency limit. In the following, it is assumed that the microphone separation is kept fixed.

2.2. DISCUSSION

Inspection of equation (12) shows that phase mismatch is most serious when the ratio of the active intensity to the mean square particle velocity is large. This condition occurs at and near velocity nodes in interference fields. Amplitude mismatch is most serious when the ratio of the reactive intensity to the mean square particle velocity is large. This also occurs in interference fields near particle velocity nodes. It follows that well-separated modes in lightly damped rooms pose a problem. On the other hand, diffuse sound fields in reverberant spaces are not particularly difficult to cope with for a measurement system with phase and amplitude mismatch. As shown in reference [18], the ratio of the mean square pressure to the magnitude of the active or reactive intensity in a reverberant room driven with wideband noise is proportional to the square root of the product of the bandwidth of the analysis and the reverberation time—and so is, by implication, the ratio of the mean square particle velocity to the magnitude of the active or reactive intensity.† Perhaps surprisingly, near fields of sources of high order do not present serious problems either, because the particle velocity takes large values under such circumstances.

Particle velocity nodes are unlikely to coincide with pressure nodes in any sound field, from which it may be concluded that the influence of phase and amplitude mismatch on the performance of sensors of total energy density is very limited, as pointed out in reference [10].

Finally, it should be mentioned that specifying the acceptable limits for amplitude and phase mismatch in terms of the residual pressure-intensity index (the ratio in logarithmic form) and the corresponding quantity for the reactive intensity is convenient, since these limits are independent of the frequency and of the microphone separation distance. This corresponds to phase and amplitude errors that are directly proportional to the frequency and to the microphone separation distance; c.f., equations (9) and (11). For example, if a sensitivity error of 0·4 dB can be tolerated at 400 Hz with a certain distance between the

† The validity of equation (12) with noise signals is not at all self-evident, since the underlying equation (6), which has been derived assuming a harmonic sound field, is valid with a band of noise only in a perfectly coherent sound field of simple structure [15]. Nevertheless, equation (12) holds good also in the general case. See Appendix B.

microphones then an error of not more than 0·1 dB would be acceptable at 100 Hz under similar conditions.

2.3. THE INFLUENCE OF PHASE AND AMPLITUDE ERRORS IN A STANDING WAVE

The gradients of the sound pressure amplitude and phase vary particularly strongly with the position in interference fields, from which it follows that such sound fields are suitable for illustrating the influence of amplitude and phase errors; cf., equation (3). The mean square pressure in a one-dimensional standing wave field can be written as

$$\overline{p^2} = A(1 + |R|^2 + 2|R|\cos(2kx + \theta)), \tag{13}$$

where R is the (complex) ratio of the reflected to the incident sound pressure and A is the mean square pressure of the incident wave (see, e.g., reference [19]). The corresponding mean square particle velocity is

$$\overline{u_x^2} = \frac{A}{(\rho c)^2}(1 + |R|^2 - 2|R|\cos(2kx + \theta)), \tag{14}$$

the (active) sound intensity is

$$I_x = \frac{A}{\rho c}(1 - |R|^2) \tag{15}$$

and the reactive intensity is

$$J_x = \frac{A}{\rho c}|R|\sin(2kx + \theta). \tag{16}$$

As can be seen, the sound field properties that control the bias errors, that is, the ratio of the active intensity to the mean square particle velocity, the ratio of the reactive intensity to the mean square particle velocity, and the ratio of the mean square pressure to the mean square particle velocity, vary with the position x more, the closer $|R|$ is to unity. An appropriate choice of $|R|$ could be 0·88, corresponding to a standing wave ratio of 24 dB, as specified in a standardized test of sound-intensity measurement systems [20].

State-of-the-art microphone sets for measurement of sound intensity exhibit phase errors of less than 0·05° above 250 Hz and amplitude errors of 0·2 dB [21]. Such equipment can pass the standing wave test for "class 1 probes" specified in reference [20] (bias errors of less than 1·5 dB), at 250 Hz with 12 mm between the microphones.

Figure 1(b) demonstrates the influence of phase mismatch as a function of the position in a standing wave field with a standing wave ratio of 24 dB, shown in Figure 1(a). It is apparent that the error is negligible except near the particle velocity minima (as also observed in references [8, 10]). It can also be seen that the sign of the phase error is important; it is more serious if the phase error and the phase angle in the sound field have opposite signs. Note that a residual pressure-intensity index of 13 dB, which corresponds to a fairly small phase error (0·16° at 250 Hz with 12 mm between the microphones, for example), paradoxically gives a much larger estimation error than a residual pressure intensity index of 8 dB (corresponding to a phase error of 0·5° at 250 Hz with 12 mm between the microphones) if the phase errors are negative. In fact, the estimated particle velocity will be zero (corresponding to an error of $-\infty$ dB) at particle velocity minima if the residual pressure-intensity index of the measurement system equals half the standing

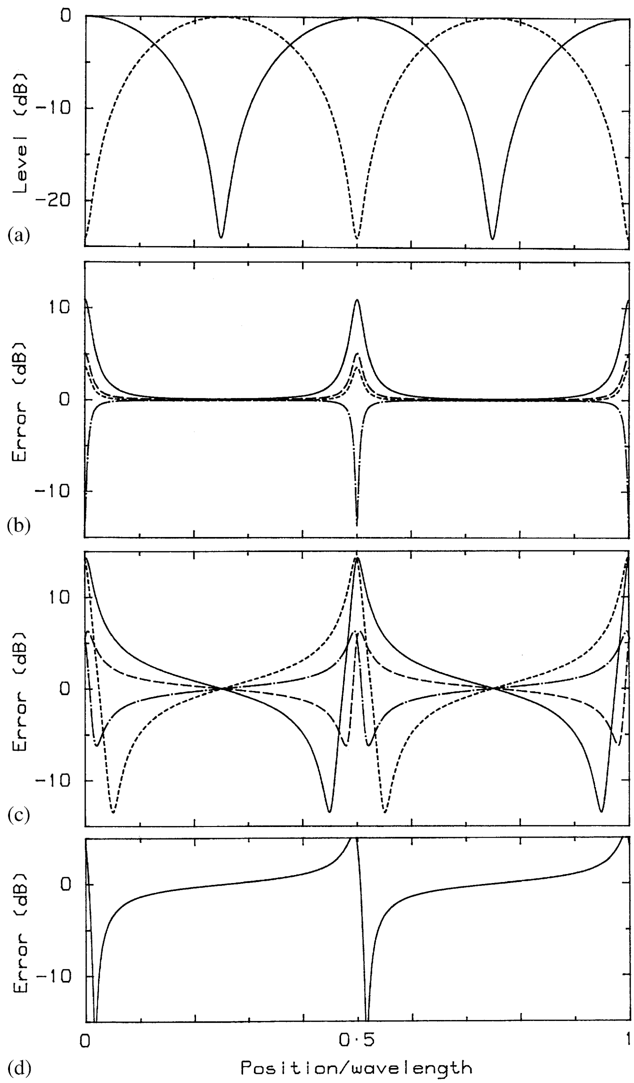

Figure 1. Measurement of particle velocity in a standing wave. (a) Standing wave pattern; ——, sound pressure level; – – –, particle velocity level. (b) Bias error due to phase mismatch as a function of the position. Residual pressure-intensity index: ——, 8 dB (positive phase error); – – –, 8 dB (negative phase error); — —, 13 dB (positive phase error); –·– ·, 13 dB (negative phase error). (c) Bias error due to amplitude mismatch as a function of the position. Residual pressure-reactive intensity index: —, 5 dB (positive amplitude error); – – –, 5 dB (negative amplitude error); ——, 10 dB (positive amplitude error); –·–·, 10 dB (negative amplitude error). (d) Bias error due to phase and amplitude mismatch as a function of the position. ——, Residual pressure-intensity index of 11 dB (negative phase error) and residual pressure-reactive intensity index of 10 dB (negative amplitude error).

wave ratio, 12 dB, and the phase error is negative. The reason is that $\hat{I}_r$ is zero under such conditions because the phase error cancels the actual phase angle in the sound field (cf., equation (8)).

Figure 1(c) shows the effect of moderate amplitude mismatch in the same standing wave field (a residual pressure-reactive intensity index of 5 dB corresponds to a sensitivity error of 0·15 dB at 250 Hz with 12 mm between the microphones; an index of 10 dB corresponds to a sensitivity error of 0·05 dB under the same conditions). With amplitude mismatch the estimation error peaks near the particle velocity minima. As can be seen, a change of sign in the amplitude error reverses the spatial pattern but does not change the error level. It is interesting to note that whereas the phase error of a well-matched microphone set as specified in reference [21] with a 12-mm microphone spacer would give rise to quite moderate bias errors in measurement of the particle velocity, the corresponding amplitude error is far more serious and affects measurements in a much larger part of the standing wave.

In practice, both amplitude and phase mismatch will occur, and some combinations of quite small mismatch errors can give very large estimation errors at some positions in the standing wave. For example, a measurement system with a residual pressure intensity of 11 dB and a residual pressure-reactive intensity index of 10 dB will underestimate the particle velocity by more than 15 dB at some positions in a standing wave with a standing wave ratio of 24 dB if the phase error is negative, as shown in Figure 1(d). (This corresponds to a phase error of 0·25° and an amplitude error of 0·05 dB at 250 Hz with 12 mm between the microphones.) The reason is that both $\hat{I}_r$ and $\hat{J}_r$ are close to zero under such conditions because the phase error cancels the actual phase angle in the sound field (cf., equation (8)) and the amplitude error cancels the actual amplitude difference in the sound field (cf., equation (10)).

According to reference [8], "$2T[=\delta_e] < 1\%$ [corresponding to amplitude mismatch of less than 0·086 dB] is certainly achievable". This is not so obvious, though. It is not an easy matter to calibrate within 0·1 dB, and the frequency response of a condenser microphone is not completely flat at low frequencies, partly because of the pressure equalization vent and partly because of the fact that the process of sound in the interior cavity tends to be isothermal rather than adiabatic, which in effect makes the air of the interior cavity of a microphone more compliant, and this increases the sensitivity. These effects are particularly important for inexpensive electret microphones, where the air stiffness of the interior cavity is a non-negligible part of the acoustic impedance of the diaphragm, and the lower limiting frequency caused by the vent tends to be higher than for measurement microphones.

3. EXPERIMENTAL RESULTS

To examine the validity of the foregoing considerations some experiments have been carried out in a standing wave tube with a standing wave ratio that varies from about 23 dB at the loudspeaker end to 24 dB near the termination. The tube, which has been constructed for testing sound-intensity probes as specified in the IEC standard [20], is made of acrylic, is 6·2 m long and has an inner diameter of 29 cm (see references [22, 23] for more details). The tube was driven at 125 Hz by a loudspeaker with a flat diaphragm with a diameter of 28 cm. A sound-intensity probe of type Brüel and Kjær (B and K) 3545 was moved through the tube with two different combinations of $\frac{1}{2}$ inch microphones, B and K 4181 and B and K 4197, in both cases with a 12-mm spacer between the microphones. The sound pressure and the particle velocity were measured using a B and K 2133 real-time analyzer.

Figure 2 shows the "true" particle velocity level as a function of the position in the tube (estimated from the measured sound pressure pattern), a prediction of a finite difference estimate of the particle velocity level determined with a measurement system with a phase error of 0·09° and an amplitude error of 0·03 dB, and the results of measuring at a number of discrete positions in the tube. The prediction, which was calculated from equations (12–16) with values of the residual pressure-intensity index (12·3 dB) and the residual pressure-reactive intensity index (9·0 dB) determined using a B and K 3541 sound-intensity

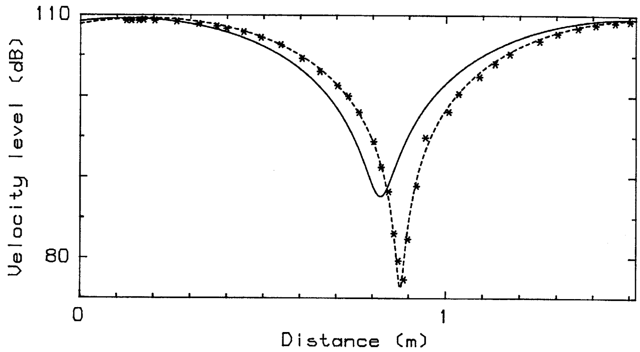

Figure 2. Particle velocity level in a standing wave tube with a standing wave ratio of 23 dB: ——, "True" particle velocity level; – – –, predicted result of measurement with a system with a residual pressure-intensity index of 12·3 dB and a residual pressure-reactive intensity index of 9·0 dB; *, experimental results.

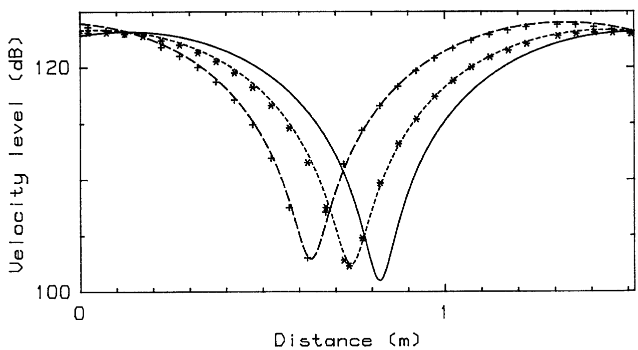

Figure 3. Particle velocity level in a standing wave tube with a standing wave ratio of 23 dB. ——, "True" particle velocity level; – – –, predicted result of measurement with a system with a residual pressure-intensity index of 19·0 dB and a residual pressure-reactive intensity index of 7·4 dB; — —, predicted result of measurement with a system with a residual pressure-intensity index of 19·0 dB and a residual pressure-reactive intensity index of 3·4 dB; *, experimental results determined with a calibrated system; +, experimental results determined with a system where the sensitivity setting in one channel has been changed 0·07 dB.

calibrator, is in almost perfect agreement with the experimental results, confirming equation (12), and confirming that the particle velocity can be seriously underestimated even with a fairly well-matched measurement system under some conditions—and not just at particle velocity nodes. It should perhaps be mentioned that the amplitude match of 0.03 dB was obtained by adjustment of the sensitivity of one of the microphone channels until the estimated particle velocity in the B and K 3541 sound-intensity calibrator assumed a minimum value in the 125 Hz one-third octave band.

Figure 3 shows a similar comparison between the "true" particle velocity, a prediction of how a measurement system with almost negligible phase mismatch (0.02°) and moderate amplitude mismatch (0·04 and 0·11 dB) would perform, and experimental results. The amplitude error of 0·11 dB was obtained by changing the sensitivity setting of one of the microphones in the analyzer from 12·5 to 12·6 mV/Pa. It is apparent that substantial errors occur. Again, there is excellent agreement between the prediction and the experimental results. However, in this case the agreement has been improved by fitting the values of the residual pressure-intensity index (19·0 dB) and the residual pressure-reactive intensity index

(7·4 and 3·4 dB). Note that amplitude mismatch of 0·11 dB is less than guaranteed by the manufacturer, indicating that amplitude matching is more critical than phase matching.

Because the measurement in a standing wave is inordinately sensitive to amplitude and phase mismatch, it seems that one can determine the residual pressure-intensity index and the residual pressure-reactive intensity index of finite difference measurement systems with much higher precision in a standing wave tube than in a coupler.

4. CONCLUSIONS

The influence of phase and amplitude mismatch on finite difference estimation of particle velocity has been examined. Amplitude mismatch is a more serious problem than phase mismatch, and the measurement principle is extremely sensitive to both kinds of mismatch errors near particle velocity nodes in interference fields, where large errors can occur even with well-matched measurement systems. However, small amplitude errors and moderate phase errors can be tolerated anywhere else. Finite difference measurements of the particle velocity will neither be affected appreciably by moderate mismatch errors in near fields nor in diffuse fields.

Finally, it should be mentioned that it has been demonstrated that one can determine the residual pressure-intensity index of a sound-intensity probe much more accurately in a standing wave tube than in a coupler.

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APPENDIX A: THE INFLUENCE OF ELECTRICAL NOISE

Finite difference estimates in frequency bands can be expressed in terms of the power and cross power spectra of the signals on which they are based (see, e.g., reference [15]). Thus, the mean square particle velocity, estimated from two bandpass filtered pressure signals $p_1(t)$ and $p_2(t)$ as follows:

$$\overline{u_r^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T\left(\int_{-\infty}^{t} \frac{p_1(\tau)-p_2(\tau)}{\rho\Delta r}\,d\tau\right)^2 \mathrm{d}t \tag{A1}$$

can also be written as

$$\overline{u_r^2} = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} S_{uu}(\omega)\,\mathrm{d}\omega = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} \frac{(S_{11}(\omega)+S_{22}(\omega)-2C_{12}(\omega))}{(\omega\rho\Delta r)^2}\,\mathrm{d}\omega, \tag{A2}$$

where S_{11} and S_{22} are the power spectra of the two unfiltered pressure signals, C_{12} is the real part of the cross-spectrum, S_{12}, and ω_1 and ω_2 are the radian band limits. If the two pressure signals are contaminated by uncorrelated noise with the power spectrum S_{nn}, the power spectrum of the particle velocity become

$$S_{uu}^n(\omega) = \frac{(S_{11}(\omega)+S_{22}(\omega)+2\mathrm{S}_{nn}(\omega)-2\mathrm{C}_{12}(\omega))}{(\omega\rho\Delta r)^2} = S_{uu}(\omega) + \frac{2S_{nn}(\omega)}{(\omega\rho\Delta r)^2}. \tag{A3}$$

The signal-to-noise ratio of a single microphone signal is obviously

$$SNR = 10\log(S_{11}(\omega)/S_{nn}(\omega)). \tag{A4}$$

It can be seen from equation (A3) that the signal-to-noise ratio of the particle velocity is

$$SNR = 10\log\frac{S_{uu}(\omega)(\rho c)^2(k\Delta r)^2}{2S_{nn}(\omega)}. \tag{A5}$$

It now follows that the signal-to-noise ratio in measurement of particle velocity is reduced by $10\log(2/(k\Delta r)^2)$ dB under conditions where $S_{11} = S_{uu}\,(\rho c)^2$.

APPENDIX B: NOISE EXCITATION

If the two pressure signals are affected by amplitude and phase mismatch, their power and cross power spectra S_{11}, S_{22} and S_{12} become

$$\hat{S}_{11}(\omega) = S_{11}(\omega)(1-\delta_e/2)^2 \simeq S_{11}(\omega)(1-\delta_e), \tag{B1a}$$

$$\hat{S}_{22}(\omega) = S_{22}(\omega)(1+\delta_e/2)^2 \simeq S_{22}(\omega)(1+\delta_e), \tag{B1b}$$

$$\begin{aligned}\hat{S}_{12}(\omega) &= (C_{12}(\omega)+\mathrm{j}Q_{12}(\omega))(1-\delta_e/2)(1+\delta_e/2)\mathrm{e}^{\mathrm{j}\varphi_e}\\ &\simeq C_{12}(\omega)-\varphi_e Q_{12}(\omega)+\mathrm{j}(Q_{12}(\omega)+\varphi_e C_{12}(\omega)),\end{aligned} \tag{B1c}$$

where C_{12} is the real and Q_{12} is the imaginary part of the cross-spectrum. Second order terms have been ignored for simplicity. It now follows that the power spectrum of the particle velocity (equation (A2)) can be written as

$$\begin{aligned}\hat{S}_{uu}(\omega) &= \frac{(\hat{S}_{11}(\omega)+\hat{S}_{22}(\omega)-2\hat{C}_{12}(\omega))}{(\omega\rho\Delta r)^2}\\ &\simeq \frac{(S_{11}(\omega)(1-\delta_e)+S_{22}(\omega)(1+\delta_e)-2C_{12}(\omega)+2\varphi_e Q_{12}(\omega))}{(\omega\rho\Delta r)^2}\\ &= S_{uu}(\omega)+\frac{(\delta_e(S_{22}(\omega)-S_{11}(\omega))+2\varphi_e Q_{12}(\omega))}{(\omega\rho\Delta r)^2}.\end{aligned} \tag{B2}$$

Since the active intensity is [24]

$$I_r = -\frac{1}{\pi}\int_{\omega_1}^{\omega_2}\frac{Q_{12}(\omega)}{\omega\rho\Delta r}\mathrm{d}\omega, \tag{B3}$$

and the reactive intensity is [15]

$$J_r = \frac{1}{\pi}\int_{\omega_1}^{\omega_2}\frac{S_{11}(\omega)-S_{22}(\omega)}{2\omega\rho\Delta r}\mathrm{d}\omega, \tag{B4}$$

it finally follows that

$$\begin{aligned}\widehat{\overline{u_r^2}} &= \frac{1}{\pi}\int_{\omega_1}^{\omega_2}\hat{S}_{uu}(\omega)\,\mathrm{d}\omega = \overline{u_r^2} - 2I_r\frac{\varphi_e}{\omega_o\rho\Delta r} - 2J_r\frac{\delta_e}{\omega_o\rho\Delta r}\\ &= \overline{u_\mathrm{r}^2} + 2I_r\frac{I_o}{\overline{p_o^2}} + 2J_r\frac{J_o}{\overline{p_o^2}} = \overline{u_r^2}\left(1+\frac{I_o}{\overline{p_o^2}/\rho c}\frac{2I_r}{\rho c\overline{u_\mathrm{r}^2}}+\frac{J_o}{\overline{p_o^2}/\rho c}\frac{2J_\mathrm{r}}{\rho c\overline{u_\mathrm{r}^2}}\right),\end{aligned} \tag{B5}$$

where ω_o is the radian centre frequency. In deriving equation (B5) it has been assumed that φ_e and δ_e are slowly varying functions of the frequency and use has been made of equations (9) and (11). Equation (B5) can be seen to agree with equation (12) to first order, demonstrating that equation (12) is valid even when equation (6) is not.